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## THE BEGINNINGS OF DEVELOPMENT.

BY PLINY EARLE CHASE,

PROFESSOR OF MATHEMATICS IN HAVERFORD COLLEGE.

*(Read before the American Philosophical Society, September 17, 1875.)*

In speculations upon the nebular hypothesis exclusive regard has usually been paid to action at the limit of possible atmosphere, or the point at which the velocity of rotation becomes equal to the velocity of revolution. Hence many popular text books state that, if the Sun were expanded until it reached the orbit of each of the planets in succession, its times of rotation would correspond with their respective times of revolution. This statement is generally understood as referring to the expansion of the *nucleus*, and with such reference it is false.

The times of rotation vary as the squares of the nucleal radius, while the times of revolution vary as the  $\frac{3}{2}$  power of the radius vector. The rotation-radius, or the radius of a nucleus which would have a rotation synchronous with orbital revolution, therefore varies as the  $\frac{2}{3}$  power of the radius vector. In my communication on "Planetary Illustrations of the Creative Fiat," I represented the rotation radii by approximate circular pendulums, the pendulum-unit being  $\frac{1}{2}$  of Sun's radius, because the centrifugal force, as Alexander has stated,\* varies as  $\frac{1}{d^3}$ ; and the distance at which the velocity acquired by infinite fall would equal orbital velocity at  $d$ , being  $2d$ ,  $\frac{1}{2^3} = \frac{1}{8}$ . The unit of orbital distance is  $\frac{3}{4}r$ , or  $(\frac{1}{8})^{\frac{2}{3}}$  of the present height of possible solar atmosphere.

In the following table, the actual values of the rotation radii for the several planetary mean distances are given, for comparison with the theoretical pendulums and for further study. An inspection of the numbers of pendulum-units shows three simple nodal groupings, with a break between Earth and Mercury, and Venus serving as a link. If we extend the nodal divisions, we find that Earth appears to have established a secondary system of its own, drawing the larger portion of the nodal material from 18 to Venus, and uniting with Sun, Venus, and Venus-Mercury in carrying the rest to Mercury.

	Theoretical.	Actual.	Difference Ratio.
Prime Multiple	504		
	112		
Neptune,†	392	392.1344	+ .0003
	112		
Uranus,	280	280.0496	+ .0002
	112		
Saturn,	168	165.8064	— .0131
	63		
Jupiter,	105	105.2344	+ .0022
	63		
Mars,	42	41.8936	— .0025

\* "Statement and Exposition of Certain Harmonies in the Solar System, by Stephen Alexander, LL.D.," (Smithsonian Contributions, 280,) p. 17.

† The names of the Planets will be used to denote their *rotation-radii* throughout the present paper, unless otherwise expressly stated. The unit of rotation-radius is  $\frac{1}{8}$

	Theoretical.	Actual.	Difference Ratio.
Earth,	12 30	30.5480	+.0183
Venus	6 } 24 } 6 } 18 } 3 } 15 } 3 } 12 } 12	23.9600	-.0017
Venus-Mercury, Mercury,	12 } 3 } 15 } 3 } 12 }	14.9913	-.0006
Half-Venus,	12		
Sun,	0		

The strongest asserter of accidental coincidences might well be staggered at such consistency of order, and the believer in universal causation may naturally ask how it is to be accounted for. I think an explanation may readily be found in the combined action of inertia and elasticity, the rhythm springing from the well-known law of harmonic densities, and therefore furnishing a strong indication of universal æthereal elasticity. I propose to inquire what harmonic series are most obvious in the general arrangement, and on what simpler and earlier nodal activities they all depend. The mathematical considerations which I shall introduce are such as belong to central forces in general, but my illustrations will all be drawn from gravitating action.

In a rotating nebula, the centre and the centrifugal unit at  $\frac{1}{3}r$ , or  $\frac{2}{3}r$  if we count from the circumference, give three nodes in the proportions 7, 8, 9, which have a common harmonic numerator in  $7 \times 8 \times 9 = 504$ . Introducing also the harmonic node  $\frac{1}{2}r = 4$ , we obtain two natural harmonic series,  $\frac{7}{9}, \frac{7}{8}, \frac{7}{7}; \frac{5}{9}, \frac{5}{8}, \frac{5}{7}$ ; etc. Now

$\frac{7}{9}$ of 504	= 392	$\Psi$	Jupiter, Earth at perijove, Earth at apojove; and Earth, Sun, Venus; repeat the ratio of Uranus to the prime multiple, $\frac{5}{9}$ . Comparing the corresponding pairs of inner and outer planets, we find $\begin{aligned} \text{♄} : \text{♀} &:: 2\frac{1}{2} : \text{♂} :: 5 : 8 \\ \oplus : \text{♂} &:: \text{♂} : \Psi :: 5 : 7 \end{aligned}$ thus introducing the second series, $\frac{5}{9}, \frac{5}{8}, \frac{5}{7}$ . If we measure the pendulums from Jupiter, Sun (105) is a mean proportional between Saturn (168-105) and Uranus (280-105). Moon and Venus repeat, in two phases, the limiting ratio of Neptune to Sun, 392. For, Moon's angular ve-
$\frac{5}{9}$ of 504	= 280	$\text{♂}$	
1st sub-harmonic	168	$\text{♂}$	
$\frac{5}{8}$ of 168	= 105	$2\frac{1}{2}$	
2d sub-harmonic	42	$\text{♂}$	
$\frac{5}{7}$ of 42	= 30	$\oplus$	
3d sub-harmonic	18	$\text{♀} \text{♄}$	
$\frac{5}{6}$ of 18	= 15	$\text{♄}$	
4th sub-harmonic	= 12		
$\frac{5}{5}$ of 12	= 12		
$2 \times 12$	= 24	$\text{♀}$	

Sun's radius. The actual rotation-radius of each planet = (rad. vec.  $\div$  18)  $\frac{3}{4}$ . E. g. Mercury's rad. vec. = 83.17 Solar-radii = 665.36 rotation units; (665.36  $\div$  18)  $\frac{3}{4}$  = 14.9913. If Sun was expanded to  $1 \pm \frac{991.3}{8}$  of its present radius, its time of rotation would equal Mercury's time of revolution.

locities of rotation and revolution being the same, we may regard her distance as a rotation-unit; and the distance of Venus's orbit from Earth's, measured in Earth's radii, corresponds with Neptune's distance from Sun, measured in Sun's radii, (6453 at mean distance, 6518 at mean aphelion). Venus's mean distance from Earth being .27667 of Earth's mean radius vector, Sun's distance is found by dividing Venus's distance by .27667.

392 × Moon's distance	= 93,155,000 miles.
6453 × Earth's radius ÷ .27667	= 92,470,000 "
6518 × " " ÷ .27667	= 93,395,000 "

Before any physical phenomenon can take place, there must be a physical force to which it can be traced. The first step in creative development should therefore be the creation of force. The potential energy of a body represents the difference between its present, or actual energy, and the greatest energy of which it is capable. In gravitation it is often referred to the results of a possible fall from the present position to the centre of attraction. If such reference were strictly true, the potential energy would always be infinite; if it is not true, it is desirable to find at what point increase of energy must cease, and all the energy must become actual. Various essays towards this determination have been made in Electricity and Chemistry; if all force is unitary in its origin, the most encouraging field for investigation would seem to be the one in which force is manifested on the largest scale—the astronomical field.

The energy which acts with reference to the Sun as a centre, is shown in two prominent ways; in planetary revolution, the velocity of which in a circular orbit I will represent by  $v_\alpha$ , and in solar rotation or retarded revolution,  $v_\beta$ . Let  $v_\gamma$  be the velocity towards which they both tend, and to which they would both be equal if all the potential energy of revolution, rotation, and internal resistance could be changed into actual energy.  $v_\alpha$  varies inversely as the square root of radius, while  $v_\beta$  varies inversely as radius, so that if the potential is expressed in units of the radius at which the velocities would become equal,  $v_\beta = \sqrt{v_\alpha v_\gamma}$ ;  $v_\beta$  always being a mean proportional between  $v_\alpha$  and  $v_\gamma$ . This, however, is not the limit of possible energy, for the velocity communicated by infinite fall =  $\sqrt{2} v_\alpha$ , and a body approaching the centre with that tangential velocity would immediately recede, never to return.  $\sqrt{2} \bar{v}_\alpha^*$  may therefore be called the velocity of dissociation.

If we suppose a circular orbit to be flattened until it becomes a linear ellipse with the solar focus at one end, the mean orbital velocity through twice the diameter =  $\frac{2}{\pi} v_\alpha$ . If shrinkage or fall continues after  $v_\beta = v_\alpha$ , the greater centrifugal force of rotation destroys rotation proper, giving

\* The superscript line denoting the greatest velocity possible.

the particles in the equatorial plane of the nucleus orbits of increasing eccentricity, until they ultimately become linear, and, when  $\frac{2}{\pi} \bar{v}_\beta = \sqrt{2} v_a$ , the velocity of dissociation is reached, and all the energy becomes actual. This velocity, as I have already shown, is the velocity of light.

If we consider Sun as a molecule in infinite space, in a trochoidal wave-stratum, every particle alternately approaches a given point and recedes, during a half-rotation. The projectile or attractive force, at or near Sun's surface, which would give this alternate approach and retreat, may be represented by gravity acting for a half-rotation,  $\frac{gt}{2}$ , which would also give the velocity of light. As the time of rotation varies inversely as gravity,  $\frac{gt}{2}$  has been, and will be constant, however much Sun may have been expanded or may hereafter contract.

In order that there may be such "mutual interchange of relations" as is needed for life and phenomenal change, there must be both resemblance and difference. There must be space and time, and also position, with some degree of fixity in space and time. A universally undulating, homogeneous æther, could manifest no variety, unless its undulations were in some way intercepted, and directed to definite points for definite purposes. There must be both elasticity and inertia, and differences of elasticity and inertia. In an expanded nebulous disc, with tendencies to nucleal aggregation at different points, those conditions would all be supplied. Every point of gross inertia, intercepting undulations from every direction would set up centripetal actions and centrifugal reactions, with tendencies to mutual compensations and equilibrium, which would give rise to physical forces in great variety.

In the second volume of Gould's *Astronomical Journal*, published in 1852, Prof. Stephen Alexander gave numerous nebular expositions, one of which treated of the Milky Way as a spiral with four branches. In the Proceedings of the Royal Astronomical Society for December, 1869, Proctor gave a paper entitled "A New Theory of the Milky Way," which also described it as being a spiral. In a paper read before the American Philosophical Society, September 20, 1872, I called attention to the following, among other facts:

"In the solar-focal parabola which passes through  $\alpha$  Centauri and has its directrix in a linear centre of oscillation of a solar diameter, twenty-seven successive abscissas may be taken in regular progression,

$$\left[ x_n = \xi^{(n^0)} \eta \pm {}^{(n^1)} \xi^{(n^2)} \right]$$

between the Star and the Sun's surface, nine of which will be extra planetary, nine will be in simple planetary relations, and nine will be intra-planetary.

"The upper extra-planetary abscissa bears nearly the same ratio to the

modulus of light, as [the limit of possible solar atmosphere] bears to solar radius.

"The limiting abscissas of the planetary series are determined by combining diametral centres of oscillation ( $2 \times \frac{2}{3}$ ), with centres of explosive condensation ( $\frac{2}{3}$ ), and of explosive oscillation ( $\frac{2}{3}$ ).

"The planetary series, between these limits, is  $\frac{1}{2} \odot$ ,  $\frac{2}{3} \oplus$ ,  $\frac{3}{4} \oslash$ ,  $\frac{4}{5}$  mean asteroid,  $\frac{5}{6} \mathfrak{L}$ ,  $\frac{7}{8} \mathfrak{h}$ ,  $\frac{7}{8} \mathfrak{O}$ .

"No probable values can be assigned to the cardinal abscissas ( $\alpha$  Centauri and  $\frac{4}{5}$  L), which will produce deviations of the theoretical from the observed values of a higher magnitude than the planetary eccentricities."

A manifest connection is thus shown between our solar system and the stellar systems, the parabolic pathway, and the relations of the modulus of light both to the solar atmosphere and to the parabolic co-ordinates, suggesting an identity of undulating and harmonic influence, which extends the significance of the first creative fiat beyond the limits of our planetary sisterhood.

We have seen that  $\frac{2}{\pi} \bar{v}_\beta = \sqrt{2} \bar{v}_\alpha$  is the limit of total dissociation, therefore  $\sqrt{\frac{2}{\pi}} v_\beta = v_\alpha$  is the limit of possible circular revolution. Planetary  $v_\alpha$  at Sun is 80.35 times as great as at Neptune;  $\sqrt{\frac{2}{\pi}} \times 80.35 = 36.18$ ; therefore at 36.18 solar radii the reactionary  $v_\beta$  bears the same ratio to  $v_\beta$  at Sun's surface, as the accelerated  $v_\alpha$  at Sun bears to  $v_\alpha$  at the outer limit of the system. This represents a rotation-period of 25<sup>d</sup>.2388, corresponding very closely with the Sun-spot estimates which have been least influenced by the unexplained acceleration\* of the spots near the equator, and differing by less than  $2\frac{1}{2}$  per cent. from the estimate which is the most reduced by allowance for that acceleration.

The range of uncertainty is as follows :

Laugier, Bianchi, and Herschel,.....	25 <sup>d</sup> .3250
"Light"-force, Theoretical.....	25.2388
Petersen.....	25.1875
Faye.....	25.0747
Lelambre.....	25.0062
Carrington.....	24.9711
Kirkwood.....	24.8259
Spörer.....	24.6245

Stockwell has found† that the mean perihelion longitudes of Jupiter and Uranus differ by exactly 180°, while the mean node longitudes of

\*I know of no attempt at explanation but the one which I have already given, based on the hypothesis that the velocity is due to combined orbital and rotational influences.

† Memoir on the Secular Variations of the Eight Principal Planets (Smithsonian Contributions, 232) p. xiv.

Jupiter and Saturn also differ by  $180^\circ$ . These accordances seem to point to a primitive nebular arrangement of alternating nucleal points, as represented in the accompanying figure.

$\Psi$  224  $\frac{1}{2}$  168  $\odot$  105  $\frac{1}{2}$  175  $\oplus$   
 └──────────┴──────────┴──────────┴──────────┴──────────┘  
 If we compare the rotation-radii for Neptune's mean aphelion and Uranus, we find the ratio of velocity from infinite fall to orbital velocity; the mean radius gives us the ratio 7 : 5; Uranus : Saturn : : 5 : 3; Jupiter : Saturn : : radius of spherical gyration : radius of homogeneous mass; the difference between Uranus and Jupiter : Jupiter : : Uranus : Saturn. The four exterior planetary orbits therefore furnish the following harmonic series :  $\frac{7}{7}, \frac{7}{5}, \frac{7}{3}, \frac{3}{3}, \frac{3}{2}, \frac{3}{5}, \frac{3}{3}, \frac{3}{5}, \frac{5}{5}, \frac{5}{3}, \frac{5}{7}$ .

Having shown that the limit of equality, from or towards which the rotating and orbital velocities of a solar equatorial particle both tend, is, like the ratio of the electric units, a quantity of the same order of magnitude as the velocity of light, let us start from that velocity, and see how nearly our results agree with those already given.

Let the velocity and time of describing radius at Sun's equator be represented by

$$\begin{aligned}
 v_a, t_a, & \text{ in solar rotation;} \\
 v_\beta, t_\beta, & \text{ in equatorial revolution;} \\
 v_\gamma, t_\gamma, & \text{ by the velocity of light;} \\
 v_\lambda & = \text{ the velocity of light;}
 \end{aligned}$$

$$\begin{aligned}
 \text{Then } t_a & \propto r^2; v_a = gt_\gamma = \frac{r}{t_a} \propto \frac{1}{r} \\
 t_\beta & \propto r^{\frac{3}{2}}; v_\beta = gt_\beta = \sqrt{gr} \propto \frac{1}{\sqrt{r}} \\
 t_\gamma & \propto r; v_\gamma = gt_a = \frac{v_\lambda}{\pi}, \text{ constant}
 \end{aligned}$$

Taking Sun's radius as the unit of length, and a second as the unit of time,  $v_\lambda = \frac{214.86}{497.83} = .4316 \text{ } r \text{ per } s$ ;  $t_\beta = \frac{365.2564 \times 86400}{2\pi (214.86)^{\frac{3}{2}}} = 1595 \text{ s.}$ ;  $v_\beta = \frac{r}{1595} = .000627r$ ;  $v_a = v_\beta^2 \div v_\gamma = .000000911r$ ; time of rotation =  $2\pi r \div v_a = 25.409 \text{ days}$ . The rotation-radii of the several planets, found by dividing the square roots of their orbital times by the square root of the time of solar rotation, are as follows :

Neptune .....	48.6693	Mars.....	5.1997
Uranus.....	34.7531	Earth.....	3.7915
Saturn .....	20.5777	Venus.....	2.9738
Jupiter.....	13.0581	Mercury.....	1.8607

These values, being given in solar radii, should be multiplied by eight

to reduce them to the centrifugal units which are given in the first table. Making the reduction, we find that the values found by the two methods differ by less than three-quarters of one per cent.

Let us take the differences between the perihelion planets of successive two-planet groups.

Neptune.....	392.1344	
Uranus.....	280.0496	
Jupiter.....	105.2344	174.8152
Earth.....	30.5480	74.6864
Sun.....	0.0000	30.5480
$\frac{1}{2}$ Venus.....	11.9800	11.9800
$\frac{1}{2}$ Mercury.....	7.4956	4.4844

If we then divide Neptune by the first difference, the first difference by the second, and so on, we get the harmonic series  $\frac{56}{25}, \frac{56}{24}, \frac{56}{23}, \frac{56}{22}, \frac{56}{21}$ ; the numerator being the quantity which is contained 9 times in the prime multiple, 7 times in Neptune, 5 times in Uranus, and 3 times in Saturn, and the greatest error in any of the theoretical denominators being less than one-half of one per cent. As the *relative* values of the rotation-radii depend on the square-roots of the orbital times, which have been determined with more precision than any other astronomical elements, these harmonies are known with great exactness.

The harmonies of which Earth forms a constituent seem, as I have repeatedly shown,\* to be more numerous than those in which other planets are exclusively involved. Is it because we are best fitted for observing things with which we are most nearly concerned, or because Earth is really of more present importance and is therefore purposely provided with more various adaptations for the nurture of intelligence than either of its sister orbs, or is it for merely æsthetic reasons, the harmonies being chords in the eternal hymn of praise which ascends from every portion of the created universe to its Creator?

A new modification of the harmonic law, in the case of Venus and Mercury, is shown, not only by the fact already mentioned, that the half-radius is introduced, (as if through a renewed operation of the relations between the radii which equalize the velocity of infinite fall and circular orbital velocity), but also by the intervention of Sun, which may perhaps be taken as an additional evidence that the parabolic connection of the solar system with its proper stellar system has produced a parabolic spiral, and may therefore be regarded as a further confirmation of Prof. Alexander's views. If we suppose, in accordance with the analogies of organic development, that the orderly processes were going on simultaneously throughout the universe, we may readily conceive that the assignment of the interior planets to their appointed places was not only the completion of our own Cosmos, but that it was also synchronous with the completion of the stellar-nebular group to which we belong.

\* Perhaps the most important of those harmonies may be the retention by Earth of one-half Sun's angular rotation energy; Sun's superficial gravity giving the velocity of light in a half-rotation, Earth's, in a whole revolution.



The connection of the two-planetary with the single-planet series, which adds to the general harmony the local harmony of equal differences on each side of the respective perihelion planets, is initiated by the relation of Uranus to Neptune, in other words by the simple harmonic which most nearly denotes the ratio of circular orbital velocity to the velocity from infinite fall. The repetition of the harmonic couplet,  $\frac{5}{7}$ ,  $\frac{5}{8}$ , both in the Jovian and in the Telluric belt, is also a consequence of the same initiative. If we look merely to the differences between the mean and the harmonic positions, Saturn and Earth are most disturbed by the action of Jupiter, Mars has fallen slightly towards Earth, Jupiter towards Saturn, Venus and Mercury towards Sun. Even the greatest differences are less than half of the mean eccentricities, so that the harmonic positions are *exactly* represented, and traversed by each planet in each orbital half-revolution. Moreover, since the geometrical mean of the actual mean radii, differs by less than  $\frac{1}{28}$  of one per cent. from the geometrical mean of the harmonic radii, the evidence of primitive harmonic influence modified by mutual perturbations, seems irresistible. Deviation within prescribed limits, allowing liberty in subordination to law, pervades all nature, and is the source of manifold supplementary harmonies and æsthetic gratifications, which would be impossible under a more rigid code.

Although the harmonic action is most simple and most striking in the rotation radii, in consequence of the greater determining influence of the nucleus, the action does not cease even after the withdrawal of all the immediate effects of nebular condensation. We accordingly find such additional rhythmical relations as are indicated by "Bode's Law," "Kirkwood's Analogy," Peirce's Phylloctactic Planetotaxy, Alexander's radial ratios, and the various accordances which I have hitherto communicated to the Society. Perhaps the most important exemplification of varied influence may be found in the mutual relations of the principal planetary masses; Neptune and Saturn being of such magnitudes as to equalize their inertia-moments near the lower nebular, or nucleal radii; Saturn and Jupiter having equal moments near the upper nebular, or vector-radii; Saturn and Uranus having equal momenta with reference to Jupiter, in the primitive arrangement of nucleal points; and Jupiter balancing Sun, in a linear pendulum, of which the geometrical mean planetary rotation radius represents a centre of oscillation, and Sun's surface represents both a centre of suspension and a fulcrum.

The first break in the Jovian belt appears to have separated the three outer planets from Jupiter, the mass of Jupiter being such as to give the same moment of inertia at a centre of spherical gyration as the remaining mass would have at the corresponding spherical surface. The outer belt subdivided in such manner that its middle planetary moment was determined by Saturn, while Saturn's was determined by Sun, the *momentum* depending on Sun, Jupiter, and Saturn, as already stated. The equality of the Saturnian and Neptunian rotation-moments completed the harmony of Chladni aggregation.

According to the latest estimates\* the masses of the four exterior planets, taking Sun as the unit, are

Jupiter.....	.0009543269	Uranus.....	.0000454545
Saturn.....	.0002855837	Neptune.....	.0000507614

the aggregate being .0013361265. The distribution of the aggregate, according to the hypothesis here given, involved the following steps:—  
 1. The square of the radius of spherical gyration being .4, in order that  $mr^2$  may equal  $m, r_1^2$  the masses must vary inversely as the square of radius. This gives .0009543761 for Jupiter, and .0003817504 for Saturn, Uranus, and Neptune. 2. Taking Saturn and Neptune as secondary centres of rotation for the remaining mass, and taking a nodal division midway between Saturn and Uranus, the Saturnian rotation-radius = 7.1402 solar radii, the Neptune-Uranian radius = 21.1508, and the masses varying inversely as radius, we obtain .0002854019 for Saturn and .0000963485 for Uranus and Neptune. 3. The equal moments of Neptune and Saturn requiring that their masses should be inversely as the squares of their rotation-radii, Neptune =  $\left\{ \frac{20.7258}{49.0168} \right\}^2 \times \text{Saturn} = .0000510257$ , leaving for Uranus .0000453228. The closeness of coincidence is shown below :

	Theoretical.	Actual.	(T-A) ÷ A.
Jupiter	9543761	9543269	+ .00005
Saturn	2854019	2855837	— .00064
Uranus	453228	454545	— .00289
Neptune	510257	507614	+ .00521
Neptune $\times 49.0168^2$		.12196	Theoretical Equality.
Saturn $\times 20.7258^2$		.12267	
Jupiter $\times 5.2028^2$		.025833	“
Saturn $\times 9.5389^2$		.025985	
Uranus $\times 48.1605$		.0021891	“
Saturn $\times 7.5715$		.0021626	
Sun $\div \frac{2}{3}$ Jupiter's r. vec.		.0013418316	“
Planetary Mass		.0013421925	
Uranus $\times 2\pi$		.0002856	“
Saturn		.0002856	
Neptune $\times \sqrt{32}$		.0002872	“
Saturn		.0002856	

I published the second theoretical equality in the 13th Volume of the Proceedings of the American Philosophical Society (p. 141), without knowing that it had ever been previously noticed, but I find, from Prof. Alexander's recent Memoir,† that he announced it to the American Association, at its Montreal Meeting, in 1857. The other nine accordances I think are entirely new. The last three introduce the following considerations :

1. If the aggregate planetary mass were at Jupiter's centre of linear oscillation, the centre of gravity of the system would be at Sun's surface.

\* For authorities, see Alexander's "Statement and Exposition," p. 3.

† Op. cit., p. 38

2. Uranus is to Saturn, as the time of describing radius in a circular orbit is to the time of orbital revolution.

3. Neptune is to Saturn, as the time of describing radius in direct fall to the centre is to the time of orbital revolution.

While thus using the convenient language of the nebular hypothesis, I have looked merely to the known laws of centripetal and centrifugal forces which are now operative, without feeling bound by any special theory. Whether planetary aggregation has sprung from gaseous or vaporous clouds, or from meteoric fall, or from explosive nuclear action, or from all combined, is immaterial; in any case the equilibrating forces would be called into play, and, if they act through the intervention of an elastic medium, the law of harmonic differences should be traceable in any resulting arrangement. "Subsidence, and the central aggregation consequent on subsidence, may go on quite as well among a multitude of discrete bodies under the influence of mutual attraction, and feeble or partially opposing projectile motions, as among the particles of a gaseous fluid."\*

Among the most important consequences of such conservation of force as is indicated by the gravity-potential and its relation to light-velocity, may perhaps be reckoned the provision which they seem to involve for the perpetuation of physical activity. In the common interpretations of the nebular hypothesis and of most of the modern thermodynamic theories, continual contraction and heat-radiation have been supposed to tend towards ultimate stagnation and universal death. In the almost exclusive regard which has been paid to centripetal influences, the increasing energy of the centrifugal force and its final preponderance have both been overlooked. To this general bias of speculative thought Prof. Alexander furnishes a weighty exception. In his Note on the origin of clusters and nebulae, he refers to appearances "as if, when they were released from superincumbent pressure, by the rupture of the outer portions of the spheroid, or other primitive form, their feeble central attraction could no longer preserve them in form; and so their centres are always broken up."† In illustration of the alternating destructive and conservative changes, he closes the Note with the following words:

"For the growing leaf is fed by the exhalations which it finds in the atmosphere; and the leaf, in its decay, nourishes the vegetating tree; the roots of that tree are embedded in the *débris* of a comparatively ancient earth; the earth itself, in view of the nebular hypothesis (of Laplace), has been detached from the sun; and the sun and other stars would now seem to be but the comparatively small fragments or drops of greater masses: the one great plan pervading the *whole*, being, BY MEANS OF A PERMITTED DESTRUCTION, TO PROVIDE FOR A MORE PERFECT ADAPTATION AND DEVELOPMENT."

\* Herschel, *Outlines of Astronomy*, § 871.

† Op. cit. p. 92.